Chapter 3: Simplified neuron and population models
The leaky integrate-and-fire neuron

\[ \tau_m \frac{d\nu(t)}{dt} = -(\nu(t) - E_L) + RL(t), \]  

(1)

\[ \nu(t^f) = \vartheta. \]  

(2)

\[ \lim_{\delta \to 0} \nu(t^f + \delta) = \nu_{\text{res}}, \]  

(3)
The leaky integrate-and-fire neuron (cont.)

A. External input $R_{I_{ext}} = 8 \text{ mV} < \text{threshold}$

B. External input $R_{I_{ext}} = 12 \text{ mV} > \text{threshold}$
The inverse of the first passage time defines the firing rate

\[ \bar{r} = \left( t^{\text{ref}} - \tau_m \ln \frac{\vartheta - RI}{v_{\text{res}} - RI} \right)^{-1} \]  

(4)
The LIF-neuron (cont.): Noise

A. Time varying input

B. Normalized histogram of ISI

C. Normalized histogram of ISI

Threshold

$C_v = 0.12$

$C_v = 0.58$
The Izhikevich neuron

\[
\begin{align*}
\frac{dv(t)}{dt} &= 0.04v^2 + 5v + 140 - u + I(t) \\
\frac{du(t)}{dt} &= a(bv - u)
\end{align*}
\]

\[v(v > 30) = c \text{ and } u(v > 30) = u - d\]
McCulloch-Pitts neuron

\[ h = \sum_{i} x_{i}^{in} \]

\[ x^{out} = \begin{cases} 
1 & \text{if } h > \Theta \\
0 & \text{otherwise} 
\end{cases} \]

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A LOGICAL CALCULUS OF THE
IDEAS IMMANENT IN NERVOUS ACTIVITY

WARREN S. MCCULLOCH AND WALTER PITTS

FROM THE UNIVERSITY OF ILLINOIS, COLLEGE OF MEDICINE,
DEPARTMENT OF PSYCHIATRY AT THE ILLINOIS NEUROPSYCHIATRIC INS
AND THE UNIVERSITY OF CHICAGO

Because of the "all-or-none" character of nervous activity, events and the relations among them can be treated by means of propositional logic. It is found that the behavior of every net can be described in these terms, with the addition of more complicated logical mechanisms containing circles; and that for any logical expression satisfaction is possible.
The firing rate hypothesis

Strech receptor on frog muscle

Firing rate [Hz] vs. Weight [g]

Tuning curve of V1 neuron in cat

Firing rate [Hz] vs. Orientation [degree]
Counter example: correlation code (?)

A. Stimulus envelope

B. Rates of individual spike trains

C. Spike-triggered rate

From DeCharms and Merzenich 1996
Integrator or coincidence detector?

A. Perfect integrator

B. Coincidence detector

From Buracas et al. 1998
Population model
Population dynamics

For slow varying input (adiabatic limit), when all nodes do practically the same, same input, etc (Wilson and Cowan, 1972):

\[ \tau \frac{dA(t)}{dt} = -A(t) + g(Rl^{ext}(t)). \] (5)

Gain function:

\[ g(x) = \frac{1}{t_{\text{ref}} - \tau \log\left(1 - \frac{1}{\tau x}\right)}, \] (6)

A. Activation function for population average in adiabatic limit

B. Activation function of hippocampal pyramidal neuron

![Graphs showing activation and discharge frequency responses.](image-url)
### Other gain functions

<table>
<thead>
<tr>
<th>Type of function</th>
<th>Graphical represent.</th>
<th>Mathematical formula</th>
<th>MATLAB implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>/</td>
<td>$g^{\text{lin}}(x) = x$</td>
<td>x</td>
</tr>
<tr>
<td>Step</td>
<td>[ \begin{cases} 1 &amp; \text{if } x &gt; 0 \ 0 &amp; \text{elsewhere} \end{cases} ]</td>
<td>[\text{floor}(0.5*(1+\text{sign}(x)))]</td>
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<tr>
<td>Threshold-linear</td>
<td>[\Theta(x)]</td>
<td>[x.<em>\text{floor}(0.5</em>(1+\text{sign}(x)))]</td>
<td></td>
</tr>
<tr>
<td>Sigmoid</td>
<td>[\int]</td>
<td>[\frac{1}{1+\exp(-x)}]</td>
<td>[1./\left(1+\exp(-x)\right)]</td>
</tr>
<tr>
<td>Radial-basis</td>
<td>[\text{exp}(-x^2)]</td>
<td>[\exp(-x.^2)]</td>
<td>[\exp(-x.^2)]</td>
</tr>
</tbody>
</table>
Fast population response!!!

Average population spike rate from IF neurons

Population dynamics (eqn 3.37)
Further Readings


Warren McCulloch and Walter Pitts (1943) *A logical calculus of the ideas immanent in nervous activity*, in *Bulletin of Mathematical Biophysics* 7:115–133.
