The leaky integrate-and-fire neuron

\[ \tau_m \frac{dv(t)}{dt} = -(v(t) - E_L) + RI(t), \] (1)

\[ v(t^f) = \vartheta. \] (2)

\[ \lim_{\delta \to 0} v(t^f + \delta) = v_{res}, \] (3)
IF simulation

A. External input $Rl_{ext} = 8 \text{ mV} < \text{threshold}$

B. External input $Rl_{ext} = 12 \text{ mV} > \text{threshold}$
IF gain function

The inverse of the first passage time defines the **firing rate**:

\[
\bar{r} = (t_{\text{ref}} - \tau_m \ln \frac{\nu - RI}{\nu_{\text{res}} - RI})^{-1}
\]
IF resistance to noise
The Izhikevich neuron

\[
\frac{dv(t)}{dt} = 0.04v^2(t) + 5v(t) + 140 - u + I(t)
\]

\[
\frac{du(t)}{dt} = a(bv - u)
\]

\(v(v > 30) = c\) and \(u(v > 30) = u + d\)
The McCulloch-Pitts neuron

\[ h = \sum_{i} x_{i}^{\text{in}} \]

\[ x^{\text{out}} = \begin{cases} 
1 & \text{if } h > \Theta \\
0 & \text{otherwise} 
\end{cases} \]

A LOGICAL CALCULUS OF THE IDEAS IMMANENT IN NERVOUS ACTIVITY

WARREN S. McCulloch AND WALTER PITTS

FROM THE UNIVERSITY OF ILLINOIS, COLLEGE OF MEDICINE, DEPARTMENT OF PSYCHIATRY AT THE ILLINOIS NEUROPSYCHIATRIC INSTITUTE, AND THE UNIVERSITY OF CHICAGO

Because of the "all-or-none" character of nervous activity, neural events and the relations among them can be treated by means of propositional logic. It is found that the behavior of every net can be described in these terms, with the addition of more complicated logical means for nets containing circles; and that for any logical expression satisfying
The firing rate hypothesis

Edgar Adrian
The Nobel Prize in Physiology or Medicine 1932
Counter example: correlation code (?)

From DeCharms and Merzenich 1996
Integrator or coincidence detector?

From Buracas et al. 1998
Population model

Temporal averaging

Population averaging
Population dynamics

For slow varying input (adiabatic limit), when all nodes do practically the same, same input, etc (Wilson and Cowan, 1972):

\[ \tau \frac{dA(t)}{dt} = -A(t) + g(R_l^{ext}(t)). \]

Gain function:

\[ g(x) = \frac{1}{t_{\text{ref}} - \tau \log(1 - \frac{1}{\tau x})}, \]

A. Activation function for population average in adiabatic limit

B. Activation function of hippocampal pyramidal neuron
# Other gain functions

<table>
<thead>
<tr>
<th>Type of function</th>
<th>Graphical represent.</th>
<th>Mathematical formula</th>
<th>MATLAB implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td></td>
<td>( g^{\text{lin}}(x) = x )</td>
<td>( x )</td>
</tr>
<tr>
<td>Step</td>
<td>[ g^{\text{step}}(x) = \begin{cases} 1 &amp; \text{if } x &gt; 0 \ 0 &amp; \text{elsewhere} \end{cases} ]</td>
<td>( \text{floor}(0.5 \times (1 + \text{sign}(x))) )</td>
<td></td>
</tr>
<tr>
<td>Threshold-linear</td>
<td>( g^{\theta}(x) = x \Theta(x) )</td>
<td>( x \times \text{floor}(0.5 \times (1 + \text{sign}(x))) )</td>
<td></td>
</tr>
<tr>
<td>Sigmoid</td>
<td>( g^{\text{sig}}(x) = \frac{1}{1 + \exp(-x)} )</td>
<td>( 1 / (1 + \exp(-x)) )</td>
<td></td>
</tr>
<tr>
<td>Radial-basis</td>
<td>( g^{\text{gauss}}(x) = \exp(-x^2) )</td>
<td>( \exp(-x^2) )</td>
<td></td>
</tr>
</tbody>
</table>
Fast population response
Further readings


Warren McCulloch and Walter Pitts (1943) A logical calculus of the ideas immanent in nervous activity, in Bulletin of Mathematical Biophysics 7:115–133.
