CSCI 1108

PID Control
Control theory

• A central ingredient for many machines are controllers that regulate some functions
• This is brought to an extreme in robotics which we have formalized with the sense-decide-act cycle
Common block diagram of an Open-loop controller

$z^*$: desired state $\rightarrow$ Controller $\rightarrow$ Plant $\rightarrow$ z: actual state

$u$: control command
Closed-loop (feedback) controller

- Controller:
  - $z^*$: desired state
  - $e$: motor error
  - $u$: control command

- Plant:
  - $z$: actual state
  - Disturbance

- Sensors:
  - Afferent
Line following
Turn at a constant rate: $\theta = \text{const}$
positive for light surface
negative for dark surface
Proportional: distance to a set-point

\[ d = 2\text{cm} \]
How can we do proportional sensing?
Turn proportional to distance (P): $\theta = c_1 x$

The rotation angle $\theta$ is getting bigger the more distant the robot is from the set point. This helps the robot staying closer to the set point. The problem is that the robot makes still large oscillations around the set point.
Turn proportional to distance (P) and change (differential) of distance (D)

$$\theta = c_1 \cdot d + c_2 \cdot \frac{dx}{dt}$$

To help dampen the oscillations we should make the turns more when we still increasing the distance to the set point and slowing it down when it gets closer to the set point. The change is described mathematically by a differential. Hence if we add such a term in the formula for the rotation angle $\theta$. 
Turn proportional to distance (P) and sum (integral) of distance (I)

\[ \theta = c_1 \cdot d + c_3 \cdot \int x \, dt \]

Another term is an integral term that is proportional to the sum of differences. While this term seems to dampen the oscillations less than the proportional term, it becomes important in some other cases, for example, if there is a gap in the line, or when the line has sharp turns.
PID controller

- Usually use a combination of all three terms
- Proportional is optimal in linear case
- Integral part is important to smoothen out rapid changes
- Differential part responds to changes in set point
- Main difficulty, adjust the weight (influence) of each control component
PID controller